

EXPERIMENTAL ANALYSIS OF SYSTEM RESPONSE

Introduction

The response of dynamical systems consists of two parts; the transient response and the steady-state response. In the transient response, the system goes from the initial state to the final state. The steady-state response of a system is the behaviour of the system as t approaches infinity. The steady-state response of a dynamical system to a sinusoidal input is called “frequency response”. The response of dynamical systems can be analyzed in either “time domain” or “frequency domain”. The transient response is generally studied in time domain using input functions such as step, impulse or ramp. Both time response and the frequency response behaviors of systems are analyzed in this experiment. So, the dynamic characteristics of these systems can be obtained and the requirements for better response can be determined.

The Analysis of System Response:

- i.) **Theoretical Way:** If the mathematical model of the system is known, the response of the system can be easily obtained using transfer functions by applying Laplace Transformation analytically.
- ii.) **Experimental Way:** If the mathematical model is not exist, the input is applied to the system and the system response is analyzed experimentally. The “step function” is applied as input in the time domain, the “sine function” is applied in the frequency domain.

The Purpose of the Experiment

RLC circuit (resistance, inductance and the capacity elements connected in series) is used in this experiment as a second order system. A step input is applied to obtain the time response and a sine function is applied for frequency response of the system. The responses of the second order system to these inputs are analyzed and they are compared with the theoretical results.

Equipment

- i.) Oscilloscope
- ii.) Signal Generator
- iii.) Resistance, inductance, capacity elements
- iv.) Connection Cables

System Model and Equation

A RLC circuit is shown in Figure 1. The transfer function of the RLC is also shown in Figure 2.

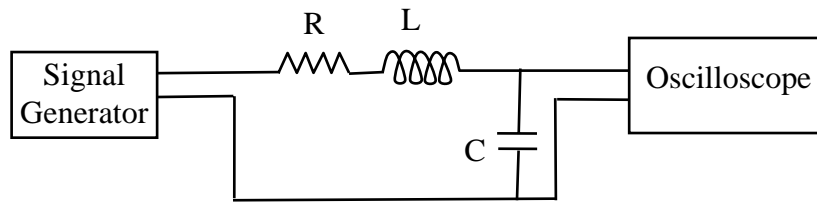


Figure 1. RLC circuit

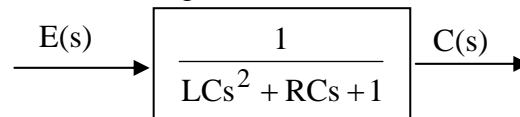


Figure 2. RLC Block diagram

Table 1. The transfer function and the Equations for RLC circuit

The transfer Function:
$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (1)$$

The natural frequency:
$$\omega_n^2 = \frac{1}{LC} \quad (2)$$

The damping ratio:
$$\zeta = \frac{R}{2} \sqrt{\frac{C}{L}} \quad (3)$$

The Experiment: Perform the steps 2 through 4 for the step input response and the steps 5 through 7 for the frequency response. Follow the other steps for theoretical analysis and comparison with experimental results.

1. Set up the circuit given on Figure-1 using the signal generator, oscilloscope, $R = \dots \Omega$, $L = \dots H$ and $C = \dots F$. Connect the circuit so that the input is displayed on CH1, and the output is displayed on CH2.
2. Switch the waveform to square wave and adjust the frequency to a low value. Adjust the horizontal setting using TIME/DIV, vertical setting using V/DIV buttons and position the input signal in the middle of screen. Assuming the square wave as a step input, apply it to the RLC circuit to obtain the step input response.
3. Change the values of R, L, and C and observe the effect of each parameter on the system response such as oscillations, steady-state error, etc. Obtain a reasonably good step response and do not change the R, L, C values for the rest of the experiment.
4. Use the Eq.2 and 3 given above to calculate the theoretical values of ω_n and ζ . Fig. 3 and the corresponding equations may be used to determine the experimental values for ω_n and ζ .
5. Switch the input waveform to Sin wave. Show the input (V_{in}) and also the output (V_{out}) on the oscilloscope screen. Get the appropriate values for the amplitude and the frequency of the sine wave in order to obtain good input tracking performance from RLC circuit ($V_{in} = V_{out}$).
6. Increase the value of the sine input from the signal generator in logarithmic way and read the amplitude of the output signal and the phase difference between the input and output signals on the oscilloscope, keep a record of your readings in Table-1.

7. Calculate the amplitude ratio $|G(\omega)|$ and the phase difference and plot the Bode diagrams (dB- ω and ϕ - ω) using Table-1.
8. Plot the Bode diagrams using the transfer function by plugging in R,L,C values.
9. Compare the theoretical and the experimental results, explain the reasons for differences.

THE STEP RESPONSE:

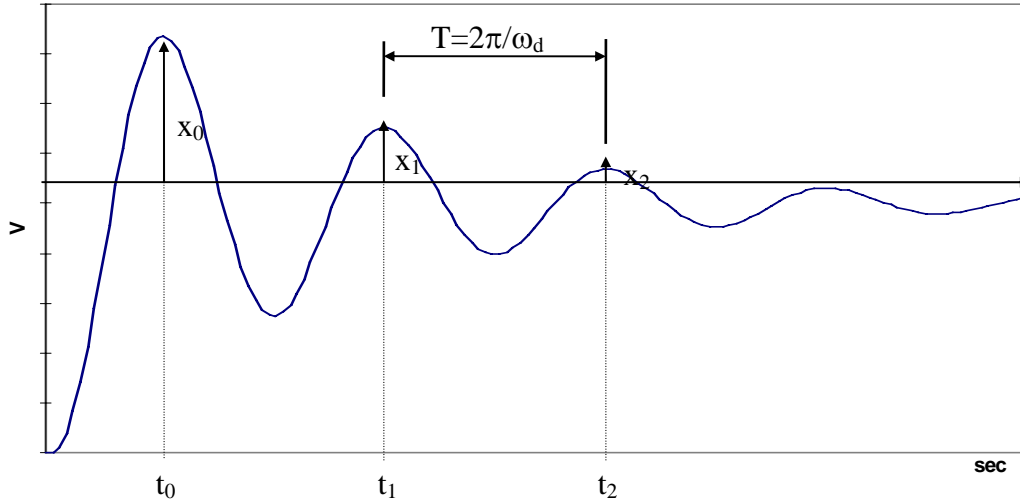


Figure 3. The step response of RLC circuit

$$\text{Logarithmic Decrement: } \frac{x_0}{x_n} = e^{\frac{2\pi\zeta n}{\sqrt{1-\zeta^2}}}, \quad \text{for } n=1 \quad \frac{x_0}{x_1} = e^{\frac{2\pi\zeta}{\sqrt{1-\zeta^2}}} \quad (4)$$

$$T = \frac{2\pi}{\omega_d} \quad (5) \qquad \omega_d = \omega_n \sqrt{1-\zeta^2} \quad (6)$$

$$t_0 = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} \quad (7) \qquad x_0 = e^{\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right)} \quad (8)$$

$$t_r = \frac{\pi - \beta}{\omega_d} \quad (9) \qquad \beta = \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta} \quad (10)$$

THE FREQUENCY RESPONSE:

The following equations are valid if $0 \leq \zeta \leq 0.707$

$$\omega_p = \omega_n \sqrt{1-2\zeta^2}$$

$$|G(\omega_p)| = 1/2\zeta\sqrt{1-\zeta^2} \quad \text{ve} \quad |G(\omega_n)| = 1/2\zeta$$

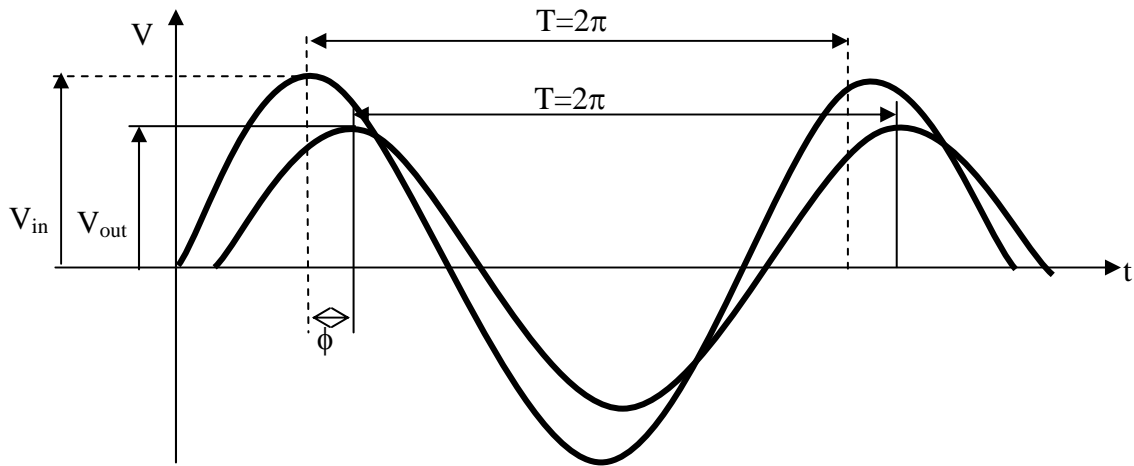


Figure 4. The frequency response of RLC circuit

The amplitude ratio can be calculated using: $|G(\omega)| = V_{out} / V_{in}$

The phase difference (in degrees) $\phi = \frac{\phi(\text{sec})}{T(\text{sec})} 360^\circ$

Table 1. R=....., C=....., L=.....

f(Hz.)	ω (rd)	ϕ (sec)	ϕ ($^\circ$)	V_{in}	V_{out}	V_{in}/V_{out}

$\omega_{\text{experimental}} = \dots$	$\zeta_{\text{experimental}} = \dots$
$\omega_{\text{theoretical}} = \dots$	$\zeta_{\text{theoretical}} = \dots$
$G(s) = \dots$	